ISSN 2229-5518

# CONSTRUCTION OF f-DIAGRAM

J.EvangelineJeba[1],Dr.Jeyabharathi[2]
[1]Department of Mathematics, Lady Doak College, Madurai
gocome7@gmail.com.

[2]Department of Mathematics, Thiagarajar Engineering College, Madurai
heart28_sweety@yahoo.co.in

**Abstract:**

*In this work we study how to construct the f-diagrams and the product of two f-diagrams and also the definition of Brauer Algebra.*

***Keyword:***

*BrauerAlgebra, f-diagram*

## 1. INTRODUCTION:

In the beginning of $20^{th}$ century invariant theorists began to study the commuting algebras of the tensor powers of defining representations for the classical groups G = Gl(n,C), Sl(n,C), O(n,C), So(n,C) and Sp(2m,C).

These algebras may be defined as follows. Let G be a classical group. Let V be its defining representation, and let $T^fV$ be the f $^{th}$ tensor power of V. (i.e.,) $T^fV = V_1 \otimes V_2 \otimes \ldots \otimes V_f$. The group action of G on V lifts to the diagonal action of G on$T^fV$ defined by $g.(V_1 \otimes V_2 \otimes \ldots \otimes V_f) = (gV_1) \otimes (gV_2) \otimes \ldots \otimes (gV_f)$. Define the commuting algebra $End_G\ (T^fV)$ of this action to be the algebra of all linear transformations of $T^fV$ which commute with this action of G. In the case of G = Gl (n, C) Schur showed that there is a surjective algebra homomorphism from $CS_f$ onto $End_{Gl\ (n,C)}(T^fC^n)$, which is an isomorphism for $f \leq n$. The kernel of this homomorphism, gives a complete explanation of the centralizer algebra
$End_{Gl\ (n,C)}(T^fC^n)$.

In 1937, when G = O(n,C) and Sp(2m,C) Richard Brauer defined two algebras $A_f^{(x)}$and $B_f^{(x)}$ where 'f' is a positive integer and 'x' is a real indeterminate. The surjective algebra homomorphism for the algebras $A_f^{(x)}$and $B_f^{(x)}$ are constructed as follows:-

$$\phi_f^{(n)}: A_f^{(n)} \longrightarrow End_{o(n,R)}\ (T^fR^n)$$

$$\chi_f^{(2m)}: B_f^{(2m)} \longrightarrow End\ sp_{(2m,R)}(T^fR^{2m})$$

If n and m are large enough then these homomorphisms are isomorphisms. When these homomorphisms are not an isomorphism then Richard Brauer failed to give the explanation of the kernel of the maps. In order to give a clear explanation of these kernels, Phil Hanlon and David Wales began to study the structure of the algebras $A_f^{(x)}$ and $B_f^{(x)}$ where 'x' is an arbitrary real. The algebras $A_f^{(x)}$ and $B_f^{(-x)}$ are isomorphic to each other. So it was only necessary to study the algebra $A_f^{(x)}$.

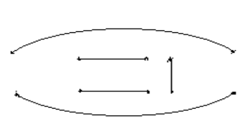
The authors were able to describe the radicals of $A_f^{(x)}$ and the matrix ring decomposition of $A_f^{(x)}$ /Rad $(A_f^{(x)})$. Later this problem was reduced to the problem of computing the ranks of certain combinatorially defined matrices $Z_{m,k}(x)$.

## 2.f-diagrams and brauer algebra

### 2.1. DEFINITION:

Let 'f' be a positive integer. An f-diagram'd' is a graph with 2f vertices and f edges such that each edge connects exactly two vertices and each vertex belongs to exactly one edge.

### 2.2. EXAMPLE:



is a 5-diagram.

## 3.Representation of an f- diagram:

An f- diagram, 'd' will be represented by a graph with 2f vertices in a plane arranged in two rows one upon the other, each of 'f' aligned vertices, the points 1,2,...,f in a top row denoted by t(d) and the points f+1, f+2,...,2f in a bottom row denoted by b(d) the vertices being labelled in the natural order from left to right.

### 3.1DEFINITION:

An edge connecting two vertices in the same row (top or bottom) will be called a horizontal edge.

An edge connecting two vertices in different rows (one in the top row and other one in the bottom row) will be called a vertical edge.

### 3.2REMARK:

In any f-diagram the number of horizontal edges in the top row is equal to the number of horizontal edges in the bottom row.

### 3.3DEFINITION:

An f-diagram'd' having no horizontal edges will correspond to a permutation $\sigma$ in $S_f$ and will be called a permutation diagram or simply a permutation.In this case the $i^{th}$ vertex of the top row will be connected to the $\sigma (i)^{th}$ vertex of the bottom row.

### 3.4NOTATION:

Let C[x] be the ring of polynomials in the indeterminate x over the field C of complex numbers. Let C(x) be the field of quotients of C[x]. Let $P_f$ denote the set of all f-diagrams. Let $V_f$ be the vector space over the field C(x) whose basis is the set $P_f$.

### 3.5DEFINITION:

Any f-diagram d in $P_f$ will have 2f vertices and f edges. There are (2f-1) possibilities to join the first vertex with any other vertex, then 2f-3 possibilities for the next one and so on.

Thus the number of f-diagrams is (2f-1) (2f-3)...5.3.1.

$$(i.e.,) \left| P_f \right| = (2f-1)(2f-3)\ldots.5.3.1.$$

### 3.6EXAMPLE:

The number of 4 diagrams is 105.

## 4.Multiplication of two elements in $v_f$ ;

We define the multiplication of any two elements in $V_f$ as follows:

Let $d_1, d_2 \in P_f$

To obtain the product $d_1 d_2$ , we proceed as follows:

**STEP 1:**

Draw $d_2$ below $d_1$

**STEP 2 :**

Connect the $i^{th}$ vertex of the top row of $d_2$ with the $i^{th}$ vertex of the bottom row of $d_1$ by a vertical dotted line.

ISSN 2229-5518

Identify the path connecting the vertices in the top row of $d_1$ and vertices in the bottom row of $d_2$.

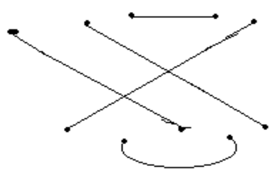
Also identify the cycles in the conjoined in diagram.

**STEP 3:**

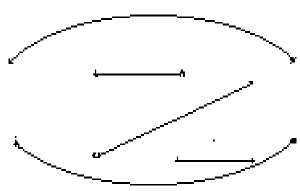
Let m be the number of cycles in the conjoined diagram obtained in step2.

If d is the diagram without cycles, then the product $d_1 . d_2$ of $d_1$ and $d_2$ is obtained by $d_1 . d_2 = x^m . d$.

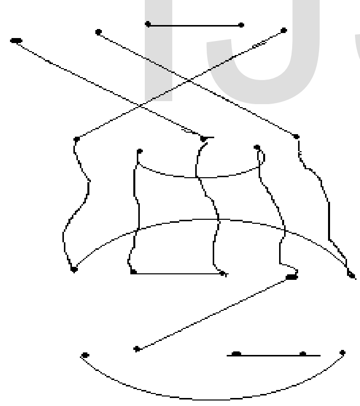
$d_1$



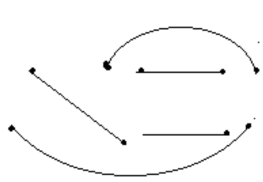
$d_2$



$d_1d_2$

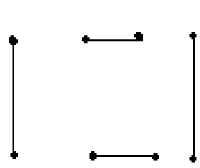


The resultant diagram is

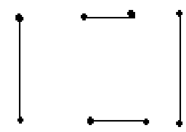


**4.1 EXAMPLE :-**

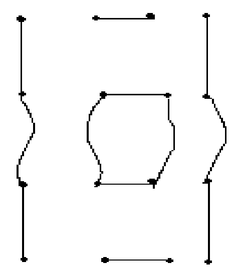
$d_1$



$d_2$

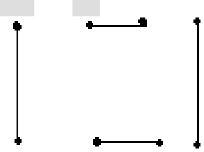


$d_1d_2$



The resultant diagram is

X.



**REMARK:**

The multiplication of two permutation diagram is compatiable with the multiplication of two permutations in $S_f$.

**DEFINITION:**

The multiplication defined above on the basis elements $P_f$ of the vector space $V_f$ is extended linearly to arbitrary elements of $V_f$ in accordance to the distributive law so as to make it into an algebra over C(x). This algebra is called Brauer algebra and will be denoted by $D_f$ (x).

**Properties of Braueralgebra $D_f$(x):**

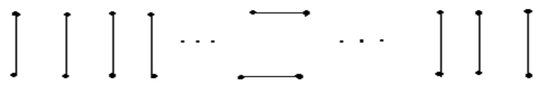
(i) $D_o = C(x) \subseteq D_1(x) \subseteq D_2(x) \subseteq \ldots$

(ii) $D_f$(x)contains the group algebra $CS_f$ as a subalgebra, $S_f$is the symmetric group on f symbols.

(iii) $D_f$(x) is a unital algebra with as the unit element.

ISSN 2229-5518

(iv) $D_f(x)$ is an associate algebra.



## 5.GENERATORS:

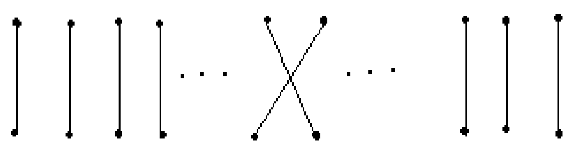
### 5.1DEFINITION:

Let $e_i$ be the f-diagram that connects the point i to i+l in the top row as well as in the bottom row and all other vertices in the top row are connected to the same vertices in the bottom row.

$e_{i =}$

i i+1

Let $g_i$be the diagram that connects the i$^{th}$ vertex of the top row with the (i+1)$^{th}$ vertex of the bottom row and (i+1)$^{th}$ vertex of the top row and to the i$^{th}$ vertex of the bottom row and to i$^{th}$ vertex of the bottom row and all other lines. connect the same points of the top



row with that of the bottom row.

$g_{i =}$

i i+1

### 5.2Properties of generators:-

$g_i^2$= 1, identify
$e_i^2$= x. $e_i$
$g_ig_j = g_jg_i$if lj-il>1
$g_ig_{i+1}g_i = g_{i+1}g_ig_{i+1}$
$e_ie_{i-1}e_i = e_i$
$e_{i-1}e_i\ e_{i-1} = e_{i-1}$
$e_ig_i = g_ie_i = e_i$
$e_ie_{i+1}\ e_i = e_i$
$e_{i+1}e_i\ e_{i+1} = e_{i+1}$
$e_ig_ig_{i+1} = e_ie_{i+1} = g_{i+1}\ g_ie_{i+1}$
$e_ie_j = e_je_i$
$e_ig_j = g_je_i$

## 6.CONCLUSION:

I have tried to give a brief sketch of some of the main ideas underlying the dynamically growing field of Brauer Centralizer Algebra. It is the natural Convergence of ideas from many areas of mathematics such as algebra, combinatorics, with those from computers science, such as algorithms, data structures. I feel confident that the current trend of studying Brauer Algebra will continue to suggest new classes of problems which are can continue for further enrichment of his knowledge.

## 7.Bibiliography:-

R.Brauer, On algebras which are connected with the semisimple continuous groups ann of math.(2) 38(1937), 857-872.
P.Hanlon and D.B. Wales, On the decompositron of Brauer's centralizer algebras, J. Algebra 121 (1989), 409-445.
P.Hanlon and D.B.Wales, Eigen values connected with Brauer's centralizer algebras, J.Algebra 121 (1989), 446-476.
IG.Macdonald, Symmetric functions and Hall polynomials, Oxford univ. press, London, 1979.
H.wenzl, On the structure of Brauer's centralizer algebras, Ann of math, 128(1988), 173-193.
I.S Luthar, I.B.S. passi, Algebra-Volume 3, Modules, Narosa Publishing house, 2005.
P.Hanlon and D.Wales, computing the discriminants of Brauer's centralizer algebras, math comput. 54. No. 19(1990), 77-0796.